AN EXPERIMENTAL DETERMINATION OF THE RESISTANCE TO THE FLOW OF OIL IN STEEL PIPES, BENDS AND ELBOWS OF SMALL DIAMETER, UNDER VARYING CONDITIONS OF VELOCITY AND VISCOSITY.

by

JOHN B. TODD, B.Sc., A.M.I.Mech. E.,
Lecturer in Engineering,
University of Edinburgh.

V O L U M E I.

I N D E X.

SECTIONS - VOL. I.

I N D E X.

TABLES - VOL. I.

23-27/

I N D E X.

TABLES. VOL. II.

I N D E X.

DIAGRAMS. VOL. III.

I. INTRODUCTORY.

Since the introduction of the Hele-Shaw and Williams-Janney variable speed transmission gears oil has been used to a very large extent as a means of transmitting power.

Such a gear consists of a pump, driven at constant speed, but having a variable stroke so that the discharge may be regulated, and a motor unit which in turn converts the power into useful work.

By varying the stroke of the pump the quantity of oil discharged may be decreased with a corresponding increase in pressure and in torque on the driving shaft of the motor.

The direction of the discharge may also be reversed so that the apparatus provides a gear which is continuously variable from its maximum speed in one direction to the maximum speed in the opposite direction.

This gear has been applied in steering gears, hoists, winches, testing machines etc. and also in the control of gun turrets and in elevating gears of ordnance.

The hydraulic motor and the pump may be a considerable distance apart and when this is the case the frictional losses in the connecting pipes become of much importance.

The experiments about to be described were carried out with a view to determining the losses which occur in straight pipes, bends and elbows of small diameter under varying conditions of velocity and viscosity of the oil used.

The aim throughout has been to obtain results under conditions which would correspond as closely as possible to those found in actual practice.

The apparatus was constructed from standard materials by/

by ordinary workshop methods, no special precautions being taken to secure, for example, specially smooth pipes or specially good joints.

Further, in order that the apparatus might be used eventually by students in the laboratory, it was arranged in such a way that experiments could be rapidly carried out.

By suitably arranging the pressure connections it was possible to switch the gauges from straight, to bend or elbow without altering the apparatus and, in order that the change from one diameter to another could be quickly made the straight pipe, bend and elbow for each size of pipe were fitted together into one unit.

2. APPARATUS.

Pipes. The pipes used were of solid drawn steel the diameters being:- 1", 7/8", 3/4", 5/8" and 3/8". For convenience in changing from one diameter to another a "test length" was made up for each size of pipe.

A "test length" is shown in Fig. 1.

It consists of a straight pipe, 10 feet in length, a right-angled bend and an elbow, with pressure connections.

The pressure connection is formed by an annular ring brazed on to the pipe, there being four small holes bored in the pipe in order that the pressure may be transmitted to the ring.

The detail of these connections is shown in Fig. 7.

At the ends of the test lengths are flanges, these being of the same size for all diameters of pipe, so that the whole test length may be readily fitted into the apparatus.

Motor. The motor used in driving the pump was 5 B.H.P. The motor and pump were mounted together on one bed-plate, the drive being transmitted by an "Oldham" flexible coupling.

HELE-SHAW PUMP.

To illustrate the action of the pump used to circulate the oil, three diagramatic views are shown in Fig. 2.

In a central tube D are four longitudinal ports. The upper two of these lead from the branch H to the space h, and the lower two from the branch G to the space g. At its outer end the central tube is fixed to the casing of the pump.

Mounted on the central tube and free to rotate round it is a cylinder body C, driven by the shaft S. In the diagram four plungers 1, 2, 3, 4 are shown in the cylinder body.

In the actual pump there are seven plungers. At the outer end of each plunger is a gudgeon pin which is held in a track T formed in a floating ring F. The gudgeon pins are free to move round the track.

The floating ring F, is carried on ball bearings in guide blocks B which are supported on guide rails L along which the floating ring may be moved horizontally to right or left of the central position by means of the control rod R.

In Fig. 2(b) the floating ring is shown in its central position and concentric with the cylinder body.

When rotation takes place the cylinder body, plungers and floating ring move round a common centre. There is no motion of the plungers relative to the cylinder body and as a consequence no pumping action.

In Fig. 2(c) the floating ring is displaced to the left. It will be seen that plunger No.1 has been moved inwards to the bottom of its stroke while No.4 has been pulled outwards.

If rotation now takes place in the direction of the arrow, plunger No.1 will be pulled outwards during the upper half revolution, and forced inwards during the lower half.

Oil is thus drawn in while the plungers are passing over the space h and is ejected while they pass the space g.

In this setting of the pump, H is the suction branch and G the delivery.

By displacing the floating ring to the right of the central position the action is reversed, G becoming the suction branch and H the delivery. It is thus possible to vary the discharge continuously from the maximum in one direction down to zero, and then up to the maximum in the opposite direction.

Since the floating ring is mounted on ball bearings and is free to rotate, the gudgeon pins although constrained to follow an eccentric path when the floating ring is displaced from the centre, actually only move in the track, a distance equal to the stroke of the pump. In this way the frictional resistance is reduced to a minimum.

Owing to the centrifugal action there is a small leakage past the plungers. This oil serves to lubricate the ball bearings and then collects in the bottom of the pump case. From this point it is returned to the tank by means of a small rotary pump. This is seen in Fig. 4 below the coupling connecting the motor and pump.

A more recent form of the Hele-Shaw pump is described in Engineering Vol. CXX. No. 3132. 1925.

---

3. GENERAL ARRANGEMENT.

The general arrangement of the apparatus is shown in Figs. 6, 7, 8.

The following description is referred to the photographs, Figs. 3 and 4.

The oil used was contained in the first instance, in the left-hand tank and in this tank there was also a cooling coil through which water could be circulated.

The motor and pump were placed at a lower level to ensure free flow of the oil into the pump and the test length of pipe was laid horizontally on the table.

By suitably adjusting the valves the oil could be circulated through the pump, the test length of pipe and back into the original tank.

The right-hand tank was carefully calibrated and, in order to measure the discharge during any test, the oil was deflected into this tank by means of a three-way cock. When the measuring tank was full, connection was made to the pump and the oil was thus returned to the original tank. The level of the oil in the measuring tank was indicated by a scale actuated by a float. This scale was graduated in inches and tenths and a vernier gave readings in hundredths.

In Fig. 4 the arrangement of the control valves is shown. The hand wheel in front of the pump controls the stroke of the pump and so the discharge. To the right of this on the discharge pipe are two valves. The first of these controls the pressure in the pump and the second is used only when pumping oil into the tank from a barrel or when emptying the system.

On the under side of the discharge pipe is a small wheel valve which connects to a spring loaded relief valve.

Above/

Above the hand wheel of the pump and at the left hand corner of the tank is a wheel valve which regulates the supply of water to the cooling coil. To the right, situated close together, are the suction cocks from the tanks and on the extreme right are shown the handle of the three-way cock and a wheel valve by means of which the pressure at discharge from the pipe may be regulated.

The pressure connections from the test length are led to the gauges which for convenience are grouped together.

All the controls are within easy reach of an operator standing in front of the gauges.

GAUGES.

The gauges are shown in Fig. 5.

For small differences in pressure a mercury U tube was used, the difference in level of the mercury columns being measured by means of a scale and vernier reading to hundredths of an inch.

When the pressure difference became too great for the mercury gauge, the cocks leading to it were closed and connection was made to the left-hand Bourdon gauge. This was graduated to 150 lb. per square inch, but to prevent risk of accident was not used above 75 lb. per sq. inch. For still greater pressures the right hand Bourdon gauge was used, pressure here being recorded up to 250 lb. per sq. inch.

In using the Bourdon gauges to measure, for example, the loss of pressure in the 10 ft. length of straight it was necessary to take two readings, one at each end of the length. The change over from one end to the other was readily accomplished by manipulation of the cocks on the pressure pipes. The scale of the left hand gauge was subdivided to read to pounds while the smallest subdivision on the right hand gauge was/

was two pounds. The gauges were specially made for the apparatus and were calibrated by dead weight test. Throughout the experiments they were checked one against the other at frequent intervals.

No trouble was experienced with these gauges and as will be seen later, the results obtained were very consistent.

On the extreme left of Fig. 5 is shown another pressure gauge on the discharge pipe. By means of this the discharge pressure could be kept constant throughout a test.

THERMOMETERS.

In order to measure the temperature of the oil two thermometers were used. These were graduated in degrees Fahrenheit.

They were placed on the permanent piping, one just before the entrance to the test length and the other just beyond the outlet from the test length. The latter is seen in Fig. 3 and a detail of the thermometer connection is shown in Fig. 7.

In calculating the results of the tests the mean of the two readings was taken.

TEMPERATURE CONTROL.

In order to vary the viscosity, the oil was heated and this was done simply by pumping. By partially closing the valve on the discharge side of the pump it was found that the oil could be heated rapidly and the range of temperature over which experiments were made was from 70°F. to 130°F.

The test length of pipe was carefully lagged with asbestos and felt covering and before commencing a test at any particular temperature time was allowed for the whole pipe to become heated up.

It/

It will be seen that the determination of temperature was not very exact.

To have kept the temperature absolutely uniform during the passage of the oil along the pipe, the latter would require to have been placed in a bath in which the temperature could be regulated and, in addition, some means of heating or cooling the oil external to the pump would have to be provided.

This would have entailed further complication and considerable expense and it was decided to adopt the simpler arrangement.

It will be seen later however, that even with the approximate temperatures obtained the results are very consistent.

By manipulating the pressure valve at the outlet from the pump and the valve controlling the supply of water to the cooling coil, it was found that the temperature could be kept practically constant for the period of any particular test.

4. METHOD OF EXPERIMENT.

It was found to be most convenient, in carrying out the experiments at any particular temperature, to complete the tests on the straight pipe at all velocities, then to do these on the bend and finally those on the elbow.

It was thought at first that it would be possible to record the pressure differences for the straight, bend and elbow by merely switching over from one to the other by means of the cocks on the gauges. The three readings would thus be obtained for one setting of the pump. This however was found to be unsatisfactory and was discarded for the following reasons:-

(a)/

(a) Owing to the resistance in the pressure connection pipes, it took a considerable time for the mercury to change from one reading to another. At first there was rapid change and then followed a period of creeping and it was difficult to determine exactly when this was completed.

(b) During the time taken for the gauge to alter, the temperature was continuously changing.

It was also found to be most satisfactory to commence any series of tests, at a particular temperature, with the full discharge of the pump and then gradually decrease to the smallest velocity obtainable, as in this way the temperature was not so liable to rise.

The lowest velocity at which tests could be made was determined mainly by the temperature conditions. When working at low temperatures it was found that, at the lower velocities it became extremely difficult to keep the temperature constant even for the short period of the test. This was due to the fact that, since only a small quantity of oil was drawn from the tank the oil remained for a considerable time in the pump, and as this was rotating at high speed the temperature rose rapidly. On the other hand at high temperatures, when only a small quantity of oil was passing along the pipe at a low velocity, the drop in temperature between the two thermometers became large.

5. DATA.

PARTICULARS OF TEST LENGTH OF PIPES.

On measuring up the pipes it was found that there was considerable variation in the diameters from the nominal values. To determine the actual diameters a pair of long, inside calipers was used so that the diameters could be measured/

measured some distance inside the pipes, thus avoiding the effect of compression of the ends due to the screwing on of the flanges. The calipers were fitted with a spring so that they could be withdrawn through the contracted portion. Six readings were made at each end of the pipe and the mean value taken.

The particulars are shown in Table I.

TABLE 1.

Nominal Diameter Inches	Actual Diameter		Area Square Feet.	Length between pressure connections. Feet.			Radius of Bend	Ratio $\frac{\text{Radius of Bend}}{\text{Radius of Pipe.}}$
	Inches	Feet.		Straight	Bend	Elbow		
1"	1"	0.0833	.00545	9.96	2.27	0.98	5.85	11.70
7/8"	$\frac{59}{64}$	0.0768	.00465	9.94	2.40	1.02	3.05	6.62
3/4"	$\frac{53}{64}$	0.069	.00374	9.94	2.42	0.98	3.40	8.21
5/8"	$\frac{41}{64}$	.0533	.002234	10.0	2.38	0.98	3.60	11.24
3/8"	$\frac{7}{16}$	0.0365	.001044	9.96	2.39	0.97	3.20	14.62

CALIBRATION OF MEASURING TANK.

The measuring tank was filled with water and this was allowed to remain until the temperature remained constant this being 61°F.

The scale reading was adjusted to 38 inches and the tank was then emptied by removing 15.6 lbs. i.e. 0.25 cu. ft. at a time, through the drain cock at the base of the tank. After each removal the scale reading was noted. Four determinations were made and the mean scale readings are given in Table 2.

TABLE 2.

Mean Scale Readings	- Inches.			
38.0	30.78	23.40	16.15	8.75
37.25	30.06	22.75	15.40	8.02
36.57	29.30	22.0	14.70	7.30
35.90	28.60	21.30	13.95	6.55
35.13	27.85	20.55	13.25	5.80
34.40	27.13	19.80	12.50	5.05
33.70	26.35	19.10	11.75	4.35
32.95	25.70	18.35	11.0	3.60
32.20	24.95	17.65	10.25	2.90
31.50	24.15	16.90	9.50	2.20

These results were then plotted, the calibration curve being shown in Fig. 10.

The portion of the scale actually used in the experiments with oil lay between 6 inches and 30 inches. These limits were fixed by the height of the suction valves in the tanks. In no case was the oil allowed to fall below the level of these valves. This precaution was taken to prevent aeration of the oil.

For/

For this reason also, it may be noted, the delivery pipes were led into the bases of the tanks. In order that the transmission of power may be efficient it is necessary that the fluid used should be practically incompressible. Prof. Hele-Shaw* has pointed out that oil loses this property very quickly when air bubbles are present. Owing to the viscosity of the oil the air bubbles can only rise slowly and the whole mass rapidly becomes saturated with air. By arranging that the oil is discharged below the surface level of the oil in the tanks this difficulty is overcome.

The calibration curve obtained from the measuring tank was a straight line and from this curve it was found that 1 inch on the float scale corresponded to a volume of 0.34 cubic feet.

Thus if:-

h = scale reading in inches per second.

Q = quantity discharged in cu. ft. per sec.

V = mean velocity in pipe in ft. per sec.

A = area of pipe in sq. ft.

$$V = \frac{Q}{A} = \frac{0.34\,h}{A} \text{ ft. per sec.}$$

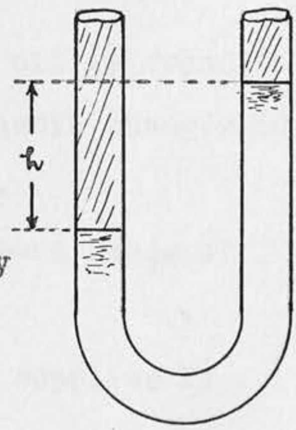
The constants thus obtained for the calculation of V are given in Table 3.

TABLE 3.

Nominal Diameter Inches	Actual Area Sq. Ft.	$V = \frac{0.34\,h}{A}$ Ft. per Sec.
1	.00545	62.35 h
7/8	.00465	73.30 h
3/4	.00374	90.90 h
5/8	.002234	152.2 h
3/8	.001044	325.7 h

* Hele-Shaw. Proc. Inst. Mech. E. 1921.

CONSTANTS FOR GAUGES.



Mercury Gauge.

If:- H = pressure in feet of water

h = mercury head in inches

S_m = specific gravity of mercury

S_o = " " " oil.

$$H = \frac{(Sm - So)\,h}{12}$$

The change in the specific gravity of the oil for the range of temperatures used in the experiments is small and, since the mercury gauge was placed at some distance from the pipes it was found to be sufficiently accurate to use the mean value of the specific gravity.

Thus :- $Sm = 13.596$

$So = 0.894$ (mean value).

and

$$H = \frac{12.70\,h}{12} = 1.058\,h.$$

Bourdon Gauges.

If:- p = pressure in lb. per sq. inch.

H = " " feet of water

$$H = \frac{p \times 144}{62.4} = 2.31\,p.$$

OIL.

For use in transmission gears mineral oil is found to be most satisfactory as it is free from organic changes to which animal and vegetable oils are liable.

For the present experiments two different kinds of oil were used.

These were both mineral oils and were supplied by the Vacuum Oil Company, Ltd.

The first oil was Gargoyle, D.T.E. Oil, Heavy Medium giving a range of Redwood viscosity from 680 seconds to 117 seconds between the temperatures 70°F. and 130°F.

In the second case Gargoyle, D.T.E. Oil, Extra Heavy was used, the range of viscosities being from 1341 seconds to 198 seconds between the same temperatures.

The total range of viscosity was thus from 1341 seconds to 117 seconds and as the values overlapped it was possible to compare the results obtained from the different samples of oil.

These oils are specially prepared to meet the most exacting conditions and possess the invaluable characteristics that they readily separate from any water with which they may become admixed in service, and that they resist the disintegrating effect of such water for extended periods.

In each case 90 gallons of oil was used for the tests.

---

6. VISCOSITY.

The absolute viscosity of a liquid is defined * as being "the tangential force on unit area of either of two horizontal planes at unit distance apart, one of the planes moving with unit velocity relatively to the other, the space between these planes being filled with the viscous substance."

This force may be expressed in dynes per square centimetre or, in British units, in poundals per square foot.

The connection between the systems of units is as follows:-

1 pound = 453.6 gms.

1 foot = 30.5 cms.

1 sq. ft. = 930.25 sq. cm.

Since Force = Mass x Acceleration

1 poundal = 1 lb. x 1 ft. per sec. per sec.

= 453.6 gms. x 30.5 cms per sec. per sec.

= 13835 dynes.

Hence.

1 poundal per sq. ft. = 13835 dynes per 930.25 sq. cm.

= 14.88 dynes per sq. cm.

KINEMATIC VISCOSITY.

In dealing with fluids in motion the kinematic viscosity is frequently used.

This is defined as being the ratio of the absolute viscosity to the density of the fluid.

If:- μ = absolute viscosity

ρ = density

ν = kinematic viscosity

then $\nu = \frac{\mu}{\rho}$ sq. ft. per sec.

* Maxwell. Phil. Trans. 1867.

COMMERCIAL TESTS FOR VISCOSITY.

Absolute viscosity is determined by finding the time required for the flow of a definite quantity of oil through a capillary tube under definite conditions. From this time the viscosity may be calculated. The operation is one of considerable difficulty and for commercial purposes, where extreme accuracy is not required, a simpler form of apparatus is used.

In Britain the viscosity is most commonly determined by means of the Redwood viscometer.

In this instrument the liquid under test, having been brought to the desired temperature is allowed to flow through an orifice of definite size in the base of the containing vessel. The time required to discharge 50 cc. is noted.

This "time of outflow" is taken as a measure of viscosity and is stated as "Redwood Seconds".

The result may also be stated as a "Redwood Number" in which case the time of outflow for the liquid under test is expressed in terms of the time of outflow for a standard fluid, usually rape oil at 60° Fahrenheit. The former method is the more common.

It has been shown* that the results obtained from such commercial tests are not proportional to the absolute viscosity of the liquid tested, and that it is not possible to obtain any very simple formula to connect the time of outflow with the absolute viscosity.

Higgins Formula.

From his experiments Higgins deduced the following reduction formulae.

If:- μ = absolute viscosity in dynes per sq. cm.

S_0 = specific gravity.

T = time of outflow in Redwood seconds.

* Higgins. Nat. Phys. Lab. = Collected Researches Vol XI, 1914

$$\mu = \left(0.0026\,T - \frac{1.715}{T}\right) So \quad \text{dynes per sq. cm.}$$

$$= \left(0.0026\,T - \frac{1.715}{T}\right) \frac{So}{14.88} \quad \text{poundals per sq. ft.}$$

The values of viscosity for the oil used in the present experiments was determined by the Vacuum Oil Company. After the oil had been circulated for some time through the system but before any tests were made, a sample of oil was forwarded to the Vacuum Oil Company and from it the Redwood viscosities were determined. In all, three determinations were made for each kind of oil, one at the beginning, one half-way through the series, and one at the end of the tests.

The values are given in Table 4 for the three tests on Gargoyle D.T.E. Oil, Heavy Medium.

TABLE 4.

Temperature °F.	Redwood Viscosity - Seconds.		
	1st Test	2nd Test.	3rd Test.
70	680	710	688
80	474	495	475
90	338	354	345
100	249	260	255
110	185	193	192
120	146	150	151
130	117	119	112

From this table it is seen that the differences are small and since the determination of the temperatures during the experiments was approximate only, the values given by the first test have been used throughout in the calculations.

In Tables 5 and 6 are given the calculated values of the absolute and kinematic viscosities.

TABLE 5.

Gargoyle D.T.E. Oil. Heavy Medium.

Temp. ^{o}F.	Weight per cu. ft. Water. lbs.	Sp. Gr. of Oil. So	Density of Oil. ρ	Redwood Viscosity T. Seconds.	Absolute Viscosity - μ Dynes per Sq. Cm.	Poundals per Sq. ft.	Kinematic Viscosity $\nu = \frac{\mu}{\rho}$
70	62.31	.9050	56.39	680	1.597	0.1073	.001904
80	62.23	.9015	56.09	474	1.108	0.0744	.001327
90	62.13	.8980	55.79	338	0.7847	0.05274	.0009452
100	62.02	.8945	55.48	249	0.5729	0.0385	.0006941
110	61.89	.8910	55.15	185	0.4203	0.02824	.0005121
120	61.74	.8870	54.17	146	0.3262	0.02192	.0004003
130	61.56	.8830	54.36	117	0.2587	0.01719	.0003162

TABLE 6.

Gargoyle D.T.E. Oil. Extra Heavy.

Temp. °F.	Weight per cu. ft. Water. lbs.	Sp. Gr. of Oil. So	Density of Oil. ρ	Redwood Viscosity T. Seconds.	Absolute Viscosity – μ Dynes per Sq. Cm.	Absolute Viscosity – μ Poundals per Sq. ft.	Kinematic Viscosity $\nu = \frac{\mu}{\rho}$
70	62.31	0.904	56.33	1341	3.1376	0.2108	.003743
80	62.23	0.900	56.01	905	2.1159	0.14221	.002538
90	62.13	0.897	55.73	634	1.4764	0.09922	.001780
100	62.02	0.893	55.39	463	1.0715	0.0720	.00130
110	61.89	0.890	55.08	330	0.7589	0.0510	.0009259
120	61.74	0.886	54.69	260	0.5929	0.03985	.0007285
130	61.56	0.882	54.30	198	0.4464	0.02999	.0005525

7. EXPERIMENTAL RESULTS.

STRAIGHT PIPES.

Gargoyle D.T.E. Oil. Heavy Medium.

The experimental results for the various pipe diameters are given in Tables 23-27. Vol. II.

While heating up the oil to the desired temperature, oil was occasionally allowed to flow through the connecting pipes between the test length and the gauges to ensure freedom from air locks.

Just before the thermometers reached the desired temperature, connection was made to the most suitable gauge and the pump was set to give maximum discharge. When the temperature was reached the three way cock was switched over and the time corresponding to a definite number of inches on the float scale was noted. During this time readings were noted from the pressure gauge and the thermometers. The pump was then set to give a lower velocity and the process repeated. This was continued down to the lowest velocity which could be obtained, having due regard to maintaining a constant temperature. To obviate errors due to discharging the oil into a cold measuring tank the latter was filled and emptied periodically during the process of heating up the oil.

In Table 23, Vol. II, are shown the results obtained on the 1-inch pipe at 70°F. It will be noticed that the temperature is somewhat variable. While the oil at this temperature is comparatively viscous and tended to heat up rapidly, the variation is mainly due to inexpert handling of the apparatus. It will be found that in the later results it became possible to maintain a much more constant temperature as the operator became more expert in manipulating the controls.

The/

The higher pressure differences were recorded in lb per sq. inch, and the lower values in inches of mercury. These were finally expressed in feet of water per foot length of pipe.

From the float gauge readings and the observed times the mean velocities were calculated. It will be noticed that the time taken for any single test was small, being in most cases under one minute, except at the lowest velocities, so that the variation in temperature was also small.

Curves.

The curves obtained for the 1-inch pipe, by plotting head lost per foot length of pipe against mean velocity are shown in Fig. 11.

The points obtained lie very closely along smooth curves.

At the lower velocities the curves are approximately straight but as the velocity becomes higher the curvature increases.

In the curves for temperatures 110°F., 120°F. and 130°F. the curvature increases rapidly to a point indicating a critical velocity, and beyond this point the curves lie close together and are approximately parallel. There is no sudden change at this criteal velocity but the change of curvature becomes more rapid as this point is approached.

Similar sets of curves are shown in Figs. 12-15. for the other diameters of pipes.

In the case of the 3/8" pipe it was not possible to reach the temperature 130°F. and at low velocities it was extremely difficult to maintain temperatures of 100° and upwards.

These portions of the curves are not reliable and the theoretical position is shown in dotted lines.

It/

It may be noted that the original curves were plotted on paper in which the inch was divided into twentieths so that the readings obtained from them were more accurate than is possible from the tracings.

Gargoyle D.T.E. Oil - Extra Heavy.

The results for this oil are given in Tables 28-31, Vol.II As this oil was much more viscous it was not possible to pump it through the 3/8" pipe at low temperatures owing to the high pressures required, so that no results were recorded for this diameter. For this reason also results are only recorded for the temperatures 100°, 110° and 120°F. for the 5/8" pipe.

The curves obtained are plotted in Figs. 16-19.

While these curves are of the same general character as before it will be seen that there is no marked indication of critical velocity until the temperature 130°F. is reached.

Values of head lost per foot length of pipe.

For convenience of reference the values of head lost per foot length of pipe, taken from the curves, have been tabulated in Tables 32, 33, Vol. II. The values for the even velocities only are given.

---

8. THEORY OF FLUID FRICTION.

From a consideration of the general case of the resistance of a body immersed in a fluid moving relatively to it, Lord Rayleigh* has shown that, on the assumptions that the resistance depends only on the linear dimensions of the body, and on the velocity, density and kinematic viscosity of the fluid, the general law may be expressed as:-

$$R = \rho V^2 \phi\left(\frac{VL}{\nu}\right) \qquad (1)$$

In this expression $\phi\left(\frac{VL}{\nu}\right)$ stands for "a function of $\left(\frac{VL}{\nu}\right)$ "

and

R = resistance per unit area of surface in absolute units.

L = linear dimension.

V = velocity.

ρ = density.

ν = kinematic viscosity = $\frac{\mu}{\rho}$ where μ = absolute viscosity

This general law also applies to the particular case of a fluid flowing through a pipe. Then if:-

R = resistance per unit area in absolute units.

d = diameter in feet.

$$R = \rho V^2 \phi\left(\frac{Vd}{\nu}\right) \qquad (2)$$

Now if for any pipe:-

l = length of pipe

h = head lost in resistance in length l .

p = pressure drop due to " " "

m = hydraulic mean depth.

i = hydraulic gradient.

then

$p = \rho h$ and $i = \frac{h}{l}$

* Rayleigh. Phil. Mag. Vol. 48. 1899.

Now Total Resistance $= R \times \pi d l$ poundals.

$$= \frac{R \times \pi d l}{g} \text{ pounds.}$$

and Total Resistance $= p \times \frac{\pi d^2}{4} = \rho h \frac{\pi d^2}{4}$

Hence. $\frac{R \times \pi d l}{g} = \frac{\rho h \pi d^2}{4}$

$$\therefore \quad \frac{R}{\rho} = \frac{h}{l} \cdot \frac{d}{4} g = i.m.g.$$

and from equation (2)

$$i\,m\,g = V^2 \phi\left(\frac{Vd}{\nu}\right)$$

or $$\frac{i\,m\,g}{V^2} = \phi\left(\frac{Vd}{\nu}\right) \qquad \text{- - - - - - - - - (3)}$$

In the assumptions made in developing the law stated in equation (2) no particular fluid is specified and thus it appears that for all fluids the values of $\frac{img}{V^2}$ will be the same for definite values of $\left(\frac{Vd}{\nu}\right)$. In other words if the conditions represented by the function $\left(\frac{Vd}{\nu}\right)$ are the same the motions of different fluids under those conditions will be similar.

The function $\left(\frac{Vd}{\nu}\right)$ may have the same value for two pipes of different diameters, but in that case the values of V or ν or both will be different and it must also be noticed that the condition of similarity involves the condition of the surface of the pipe. If pipes of different diameters are to be compared the roughness of the surfaces should vary in proportion to the diameters for the condition of similarity to be true.

To investigate the effect of roughness of surfaces Heywood* has recently carried out experiments on the flow of water in pipes and channels.

* Heywood. Proc. Inst. C.E. Vol. 219. 1925.

In dealing with the flow of oil however, the velocities used in practice will rarely exceed the critical velocity. Below this velocity, stream line flow exists and it follows that the roughness of the pipe has no influence, beyond the fact that a very rough pipe may be expected to cause a change from stream line to turbulent flow at a lower velocity than that at which it would occur in a smooth pipe.

9. STREAM LINE FLOW.

Poiseiulle, by his experiments on flow in capillary tubes deduced the following law* for stream line flow:-

$$\text{Loss of pressure} = \frac{32\mu l V}{d^2} \quad - - - - - (4)$$

and if R = resistance per unit area

$$\text{Total resistance for a length } l = R \times \pi d l = \frac{32\mu l V}{d^2} \times \frac{\pi d^2}{4}$$

$$\therefore R = \frac{8\mu V}{d}$$

and since $\nu = \frac{\mu}{\rho}$

$$\frac{R}{\rho V^2} = \frac{8\nu}{V d} \quad - - - - - - - - - - - - (5)$$

Comparing this with equation (2) it is seen that, for stream line flow the function $\left(\frac{Vd}{\nu}\right)$ must be equal to $\frac{8\nu}{Vd}$.

If now, values are given to $\left(\frac{Vd}{\nu}\right)$ the corresponding values of $\frac{R}{\rho V^2}$ may be calculated and, if the latter are plotted as ordinates against the former a curve, representing the conditions for stream line flow is obtained, and this is applicable to all fluids.

The form of this curve is shown in Fig. 9.

* Hydraulics and its Applications. Gibson.

The values for plotting this curve are tabulated in Table 7.

TABLE 7.

$\frac{Vd}{\nu}$	$\frac{R}{\rho V^2}$	$\frac{Vd}{\nu}$	$\frac{R}{\rho V^2}$
100	·080	1600	·0050
150	·0533	1700	·0047
200	·040	1800	·00444
300	·0266	1900	·00421
400	·020	2000	·0040
500	·016	2100	·00381
600	·0133	2200	·00364
700	·0114	2300	·00348
800	·010	2400	·00333
900	·0088	2500	·0032
1000	·008	2600	·00308
1100	·00727	2700	·00296
1200	·00666	2800	·00286
1300	·00615	2900	·00276
1400	·00571	3000	·00267
1500	·00533		

To verify the theory stated above Stanton* carried out an extended series of experiments on smooth-drawn brass pipes of diameters varying from 0.3 cm. to 10 cm. The lengths tested were without joints and before reaching the test portion of the pipe the fluid was allowed to pass through a sufficient length to ensure that the flow was not disturbed by influences other than those due to the pipes themselves.

The/

* Stanton & Pannell. Nat. Phys. Lab. Collected Researches. Vol. XI. 1914.

The fluids used in these experiments were air, water and thick oil. The results show that below the critical velocity the points lie closely round the theoretical curve. With the oil the critical velocity was not reached but with the air and water it was found to occur when $\frac{Vd}{\nu}$ had the value 2500.

Beyond this point there is a sudden rise in the values of the ordinates $\frac{img}{V^2}$ and then these values decrease gradually throughout the range of the experiments.

COMPARISON OF PRESENT EXPERIMENTS WITH THEORETICAL CURVE.

In order to compare the present experimental results with the theoretical values Tables 34-37, Vol. II, have been prepared for Gargoyle, D.T.E. Oil, Heavy Medium.

Taking the values of head lost per foot length of pipe for even velocities as in Table 32. Vol. II, the values of $\frac{img}{V^2}$ and $\frac{Vd}{\nu}$ have been calculated.

For the 1-inch pipe these values are shown in Table 34 Vol. II.

The theoretical curve is shown by the full line in Fig. 20. This is plotted from the values given in Table 7.

The experimental points obtained are plotted for all diameters and all temperatures. These points are closely and fairly uniformly grouped round the theoretical curve for the lower values of $\frac{Vd}{\nu}$.

The only exceptions are certain values for the 3/8" pipe and these values were obtained with low velocities at temperatures of 110°F. and 120°F. It has already been pointed out that exact temperature control was impossible under these conditions with the existing apparatus, and this accounts for the discrepancies in these results.

A similar curve is plotted in Fig. 21 to show the results obtained/

obtained with Gargoyle D.T.E. Oil Extra Heavy, the values being given in Tables 39-41. Vol. II.

10. CRITICAL VELOCITY.

A change in the condition of flow is indicated on both of these diagrams, Figs. 20 and 21, at a value of $\frac{Vd}{\nu}$ of approximately 1200. This is very much lower than the value given by Stanton, viz. 2500.

Durand* gives the value at which the critical velocity occurs as varying from 2000 to 2500 but no details are given of the conditions under which the values were obtained.

In the present experiments the aim has been to obtain results which would correspond with those likely to occur under practical conditions. The condition under which the flow occurs, in the experimental pipes used, was bad, probably worse than may be expected in a normal practical installation of good design.

It will be seen by reference to Fig. 4 that the oil after leaving the pump and before reaching the test length, passes through a valve, a Tee piece, two right angled bends and two joints. These all occur in a length of 4 ft. 6 inches.

The length between the second joint and the first pressure connection on the test length is only one foot, so that disturbance due to these resistances is likely to be shown at comparatively low velocities. The system of piping adopted was fixed mainly from a consideration of the floor space available in the laboratory and because such a system gave a very convenient arrangement of controls so that the apparatus could be handled by a single operator.

* Durand. Hydraulics of Pipe Lines.

While it is unlikely that so many resistances will occur in such a short length of piping in most cases in practice, it will frequently happen that there are several bends and joints between the pump and the motor unit and in all cases there will be a considerable amount of vibration from the pump itself. The pump runs at high speed, in the present instance at 1000 revs. per min. and, as there are seven plungers each making two strokes per revolution it will be seen that the number of impulses given to the oil is very high.

This vibration was not noticeable to the eye, on the pipes themselves but could be distinctly seen in the mercury columns of the thermometers.

Osborne Reynolds pointed out the existance of two critical velocities in a pipe, the lower of which corresponded to the velocity at which the character of the flow changed from turbulent to stream line. In the present case each series of tests was commenced at the highest velocity which was then gradually reduced so that in all cases the lower critical velocity was likely to be obtained.

The expression $\frac{Vd}{\nu} = 1200$ - - - - - - - - - - - (6)

may then be expected to give the critical velocities under conditions where there are several factors likely to cause disturbance of the flow.

The values obtained from it may be looked upon as the lower end of a scale of values, the upper end of which would be obtained by using the figure 2500 given by Stanton, the value to be selected in any particular case depending on the conditions obtaining in the pipe line.

Referring again to Figs. 20 and 21 it is seen that for both kinds of oil the points lie closely grouped round the theoretical/

theoretical curve for values of $\frac{Vd}{\nu}$ below 1200. Thus for given values of V and d the behaviour of each kind of oil is the same when the viscosity is the same. This is also seen by comparing Tables 34 and 39, Vol. II. The Heavy Medium Oil at 90°F. has a Redwood Viscosity of 338 secs., while the Extra Heavy Oil at 110°F. has a Redwood Viscosity of 330 seconds, and the values of head lost per foot length are in very close agreement.

In The following Table the values of the critical velocities, for both kinds of oil are given, arranged in order of viscosity. These have been calculated from equation (6) using the actual diameters of the pipes.

TABLE 8.

Nom. Diam.	1"	7/8"	3/4"	5/8"	3/8"
Redwood Viscosity.	Critical Velocity - Feet per Second.				
1341	53.9	58.5	65.0	83.3	123.5
905	36.5	39.6	44.1	57.1	83.1
680	27.4	29.7	33.1	42.9	62.7
634	25.6	27.8	31.0	40.8	58.5
474	19.1	20.8	23.1	29.9	43.6
463	18.8	20.3	22.6	29.3	42.8
338	13.6	14.8	16.5	21.3	31.1
330	13.4	14.5	16.1	20.8	30.5
260	10.5	11.4	12.7	16.4	24.0
249	10.0	10.9	12.1	15.6	22.8
198	8.0	8.7	9.6	12.4	18.2
185	7.4	8.0	8.9	11.5	16.9
146	5.8	6.3	7.0	9.0	13.2
117	4.6	4.9	5.5	7.1	10.4

11. CALCULATION OF LOSS OF HEAD.

The formula most commonly used for the calculation of the loss of head is

$$h = \frac{4 f l}{d} \cdot \frac{V^2}{2g} \quad \text{- - - - - - - - - - - - - -} \quad (7)$$

where f is a constant determined by experiment.

As the constant f forms a convenient means of presenting the variation in frictional resistance to engineers who are accustomed to using the above formula, the values obtained in the experimental results are shown in Tables 34-42. Vol. II.

It may be pointed out here that there is a definite connection between this formula and the theoretical formula already given in equation (3).

Equation (7) may be written in the form

$$f = \frac{h}{l} \cdot \frac{d}{4} \cdot \frac{2g}{V^2} = 2 \frac{img}{V^2} \quad \text{- - - - - - - - -} \quad (8)$$

and since from equation (3)

$$\frac{img}{V^2} = \phi\left(\frac{Vd}{\nu}\right)$$

it follows that the so-called constant f is a function which depends on the velocity and viscosity of the fluid and on the diameter of the pipe.

For stream line conditions of flow, where the roughness of the pipe has no influence, f may evidently be obtained directly from the theoretical formula. It will be shown later however that it is not necessary to calculate f, as the loss of head may be obtained directly in a simple manner.

For turbulent conditions this procedure would not be sufficiently accurate since the roughness has considerable influence, and until further information is obtained regarding the/

the effect of roughness, recourse must still be had to experiment to determine appropriate values of f for a given set of conditions.

In the tables f has been calculated from equation (8). The position of the critical velocities is indicated by horizontal lines as for the 1-inch pipe in Table 34 Vol. II for temperatures 110°, 120° and 130°F.

It will be seen from Fig. 20 that the plotted points corresponding to the values below these lines lie above the theoretical curve, indicating turbulent flow.

LOSS OF HEAD FROM THEORETICAL FORMULA.

It has been shown in equations (2) and (3) that

$$\frac{R}{\rho V^2} = \phi\left(\frac{Vd}{\nu}\right) = \frac{img}{V^2}$$

and in equation (5) that, for stream line flow

$$\frac{R}{\rho V^2} = \frac{8\nu}{Vd.}$$

Hence. $$\frac{img}{V^2} = \frac{8\nu}{Vd.}$$

and $$i = \frac{8\nu V}{m.g.d.} \quad \text{- - - - - - - - - - - -} \quad (9)$$

But $m = \frac{d}{4}$ and $i = \frac{h}{l}$ and thus if h is the head lost per 100 feet of pipe

$$h = \frac{3200\,\nu\,V}{gd^2} \quad \text{- - - - - - - - - - - -} \quad (10)$$

Where h = head lost per 100 feet of pipe

d = diam. of pipe in feet.

V = velocity in ft. per sec.

ν = kinematic viscosity $= \frac{\mu}{\rho}$

In order to use this formula the kinematic viscosity must first of all be obtained from the Redwood viscosity by the method indicated in pp. 16 and 17.

12. APPROXIMATE FORMULA FOR LOSS OF HEAD.

For many practical purposes extreme accuracy is not necessary and indeed, as will be shown later in discussing the influence of diameter, is not possible of attainment.

Having this in view, the formula in equation (10) may be simplified.

If T = Redwood viscosity in seconds.

μ = absolute viscosity.

S_o = specific gravity of oil.

Higgins' Formula gives

$$\mu = \left(0{\cdot}0026\ T - \frac{1{\cdot}715}{T}\right) \frac{S_o}{14{\cdot}88}$$

and as an approximation this may be written

$$\mu = 0{\cdot}0026\ T \times \frac{S_o}{14{\cdot}88} \quad \text{- - - - - - - - - (11).}$$

Now.

Kinematic viscosity $= \nu = \frac{\mu}{\rho} = \frac{\mu}{S_o \times w}$ where w = weight of 1 cubic foot of water and hence from equation (11)

$$\nu = \frac{{\cdot}0026\ T \times S_o}{14{\cdot}88} \times \frac{1}{S_o \times w} = \frac{{\cdot}0026\ T}{14{\cdot}88 \times w}.$$

The variation of w with change of temperature is small and if this be neglected equation (10) becomes

$$h = \frac{3200\ V}{32{\cdot}2\ d^2} \times \frac{{\cdot}0026\ T}{14{\cdot}88 \times 62{\cdot}4}$$

$$= 0{\cdot}000278\ \frac{V\ T}{d^2} \quad \text{- - - - - - - - - (12)}$$

in which h, the loss per 100 ft. of pipe is expressed directly in terms of velocity, Redwood viscosity and diameter.

----

13. APPROXIMATE FORMULA FOR CRITICAL VELOCITY.

It has been shown that for the conditions under which the present experiments have been carried out, the change from stream line to turbulent flow occurs when

$$\frac{Vd}{\nu} = 1200 \quad - - - - - - - - - - - - - \quad (6).$$

This formula involves the use of the kinematic viscosity ν, the calculation of which is troublesome since the absolute viscosity and the density are required.

For many practical purposes the following approximate formula will be sufficiently accurate.

From equation (11) et seq.

$$\nu = \frac{.0026\ T}{14.88 \times w}$$

where w is the weight of 1 cubic foot of water.

If the variation in the value of w be neglected then

$$V_c = \frac{1200 \times .0026\ T}{14.88 \times 62.4 \times d} = 0.00336\,\frac{T}{d}. \quad - - - - \quad (13)$$

Where.

T = Redwood viscosity in seconds.

d = diameter of pipe in feet.

V_c = critical velocity in ft. per sec.

COMPARISON OF THEORETICAL AND APPROXIMATE FORMULAE.

In the following table the head lost per 100 feet of pipe (h) has been calculated in terms of the velocity (V) taking values of viscosity and diameter at random from both sets of tests. These are arranged in order of viscosity.

TABLE 9.

Redwood Viscosity T.	Kinematic Viscosity	Nominal Diameter Inches.	d^2 (actual)	Values of h in terms of velocity: Theoretical. $h = \frac{3200}{32.2}\frac{\nu V}{d^2}$	Values of h in terms of velocity: Approximate. $h = .000278\frac{VT}{d^2}$	Error %
1341	.003743	7/8	.005898	63.07 V	63.19 V	+ 0.19
680	.001904	1	.00694	27.27 V	27.27 V	0
634	.00178	3/4	.00476	37.15 V	37.05 V	- 0.27
463	.00130	5/8	.00284	45.48 V	45.35 V	- 0.29
338	.0009452	7/8	.005898	15.93 V	15.94 V	+ 0.07
330	.0009259	7/8	.005898	15.6 V	15.57 V	- 0.19
249	.0006941	3/8	.00133	51.8 V	52.0 V	+ 0.38
198	.0005525	1	.00094	7.91 V	7.94 V	+ 0.38
185	.0005121	3/4	.00476	10.70 V	10.80 V	+ 0.90
146	.0004003	5/8	.00284	14.0 V	14.29 V	+ 2.08
117	.0003162	1	.00694	4.53 V	4.69 V	+ 3.5

From this table it is seen that the errors involved by using the approximate formula are small. For a change of viscosity from 1341 seconds to 185 seconds the error is in all cases less than 1 per cent. At the lowest value of the viscosity, 117 seconds, the error is 3.5 per cent.

COMPARISON WITH EXPERIMENTAL RESULTS.

In order to compare the calculated values of head lost with those obtained experimentally, Table 10 has been prepared.

Here the head lost per 100 feet of pipe, expressed in feet of water, has been determined.

(a) from the theoretical formula $h = \frac{3200}{32.2} \frac{\nu V}{d^2}$

(b) " " approximate " $h = .000278 \frac{VT}{d^2}$

(c) " " experimental results.

Values of viscosity to cover the whole range of the experiments have been taken.

TABLE 10./

TABLE 10.

Redwood Viscosity Seconds.	Nominal Diameter Inches.	Mean Velocity Ft. per Sec.	Head lost per 100 ft. of pipe.			Difference. Exp. & Theor.
			Theoretical Formula. Ft. of Water.	Approximate Formula. Ft. of Water.	Experimental. Ft. of Water.	%.
1341	7/8	2	126	126	126	0
		4	252	252	253	+ 0.4
		6	378	379	385	+ 1.85
		8	504	505	517	+ 2.57
		10	631	632	652	+ 3.32
		12	756	758	790	+ 4.30
		14	883	885	927	+ 4.98
		16	1010	1012	1071	+ 5.90
680	1	2	54.5	54.5	52.9	- 2.9
		4	109	109	106	- 2.7
		6	163	163	156	- 4.3
		8	218	218	208	- 4.5
		10	273	273	262	- 4.0
		12	327	327	321	- 1.9
		14	382	382	392	+ 2.6

TABLE 10. (continued).

Redwood Viscosity Seconds.	Nominal Diameter Inches.	Mean Velocity Ft. per Sec.	Head lost per 100 ft. of pipe. Theoretical Formula Ft. of Water	Head lost per 100 ft. of pipe. Approximate Formula. Ft. of Water.	Head lost per 100 ft. of pipe. Experimental. Ft. of Water.	Difference. Exp. & Theor. %
634	3/4	2	74.3	74.1	74.0	- 0.40
		4	148.5	148	147	- 0.10
		6	223	222	220	- 1.34
		8	297	296	295	- 0.68
		10	371	370	369	- 0.68
		12	446	445	444	- 0.45
		14	520	519	517	- 0.57
		16	594	593	593	- 0.17
		18	669	667	668	- 0.15
		20	743	741	746	+ 0.40
		22	818	814	827	+ 1.10
463	5/8	2	90.9	90.6	90.0	- 0.91
		4	182	181	177	- 2.7
		6	273	272	265	- 2.9
		8	363	362	354	- 2.5
		10	455	454	445	- 2.2
		12	545	544	535	- 1.83
		14	636	635	626	- 1.57
		16	726	725	718	- 1.10
		18	817	816	810	- 0.85
		20	909	906	903	- 0.66

TABLE 10. (continued).

Redwood Viscosity Seconds.	Nominal Diameter Inches.	Mean Velocity Ft. per Sec.	Head lost per 100 ft. of pipe. Theoretical Formula Ft. of Water.	Approximate Formula Ft. of Water	Experimental Ft. of Water.	Difference Exp. & Theor. %
338	7/8	2	31.9	31.9	31.5	- 1.0
		4	63.8	63.8	63.5	- 0.3
		6	95.6	95.6	96.0	+ 0.4
		8	127.5	127.5	128	+ 0.39
		10	159	159	162	+ 1.9
		12	191	191	198	+ 3.7
		14	223	223	234	+ 4.9
		16	255	255	273	-
249	3/8	2	104	104	90.0	- 1.4
		4	207	208	182	- 0.7
		6	311	312	280	- 9.9
		8	415	416	379	- 8.7
		10	518	520	482	- 6.9
		12	621	624	590	- 5.0
		14	725	728	698	- 3.7
		16	828	831	805	- 2.8
		18	931	936	919	- 1.3
		20	1040	1040	1032	- 0.76

TABLE 10. (continued).

Redwood Viscosity Seconds.	Nominal Diameter Inches.	Mean Velocity Ft. per Sec.	Head lost per 100 ft. of pipe. Theoretical Formula Ft. of Water	Approximate Formula. Ft. of Water.	Experimental. Ft. of Water.	Difference. Exp. & Theor, %
185	3/4	2	21.4	21.8	21.0	- 1.9
		4	42.7	43.1	42.5	- 0.47
		6	64.2	64.8	65.0	+ 1.25
		8	85.5	86.4	89.0	+ 4.1
		10	107	108	114	-
146	5/8	2	28.0	28.6	27.0	- 3.6
		4	56.0	57.2	55.0	- 1.8
		6	84.0	85.8	80.0	- 4.8
		8	112	114	108	- 3.6
		10	140	143	140	-
117	1	2	9.06	9.38	8.8	- 2.9
		4	18.1	18.8	19.0	+ 4.9
		6	27.2	28.2	31.5	-
		8	36.2	37.5	58.0	-
		10	45.3	46.9	96.0	-

Where the results recorded in the above tables exceed the critical velocity the position of the latter is indicated by a horizontal line.

It will again be seen that the values of head lost as obtained by the theoretical and approximate formulae are in very close agreement over a wide range. As is to be expected the experimental results vary considerably, the greatest differences occurring in the 3/8 inch pipe where the conditions were particularly difficult.

13.[a] INFLUENCE OF DIAMETER.

Reference to Table 1 shows that the actual diameters of the pipes vary considerably from the nominal values.

In practice, calculations will most commonly be made using the nominal diameter, since the variation in diameter is likely to be irregular and the determination of the mean diameter for a system of piping consisting of many lengths of pipe, would be a matter of some difficulty. In practical design also the actual pipes to be used will rarely be available for measurement.

In order to investigate the effect of using the nominal diameter one set of results has been re-calculated.

For this purpose the 7/8 inch pipe has been taken, in which the actual diameter was $\frac{59''}{64}$ i.e. + 5.35%.

From Table 24, Vol. II, the experimental results at 90°F. are obtained and the velocities are re-calculated, giving the values shown in Table 11.

TABLE 11./

TABLE 11.

Tests on 7/8 inch pipe calculated from nominal diameter.

Tank Gauge. Inches per Sec.	Mean Velocity. Ft. per Sec.	Head lost per foot length. Ft. of Water.
.2366	19.3	2.98
.2018	16.4	2.49
.1655	13.48	2.01
.1452	11.81	1.70
.1192	9.73	1.41
.1038	8.45	1.21
.0862	7.02	1.0
.0664	5.40	0.761
.0498	4.06	0.562
.02318	1.88	0.25

The curve corresponding to these values is shown by a chain-dotted line in Fig. 12. It will be seen that the effect of using the nominal diameter is to move all the points to the right so that any particular value of head lost apparently occurs at a higher velocity than is actually the case.

It follows that if the value of head lost is read off from this curve for any velocity, it will be less than would actually obtain in the pipe at that velocity.

These apparent values of head lost and the corresponding values of $\frac{img}{V^2}$ and $\frac{Vd}{\nu}$ are given in Table 12.

TABLE 12./

TABLE 12.

Temperature ^{o}F.	Mean Velocity V Ft. per Sec.	Apparent Loss of Head i	$\frac{1}{V^2}$	$\frac{\mathrm{img}}{V} = 0.887 \frac{1}{V^2}$	$\frac{Vd}{\nu}$
	2	0.27	.0675	.0396	154
	4	0.55	.0344	.0202	309
90^{o}	6	0.84	.0233	.0137	463
$\nu = .0009452$	8	1.14	.0178	.01045	617
$\frac{Vd}{\nu} = \frac{.0729}{.0009452} V$	10	1.44	.0144	.00845	771
$= 77.13$ V	12	1.76	.0122	.00716	926
R.V. 338 Secs.	14	2.09	.01066	.00625	1080
	16	2.43	.00949	.00557	1234

When these points are plotted on the theoretical stream line curve, as shown by the chain-dotted line in Fig. 20 it is found that they lie on a curve, parallel to the theoretical curve but below it, the difference in this case being due to under-estimation of the diameter.

If then, experimental values are to be compared with the theoretical curve it is essential that the actual diameter be accurately determined.

EFFECT OF USING NOMINAL DIAMETER IN THEORETICAL FORMULA.

In order to show the effect of using the nominal diameter when calculating the loss of head from the theoretical formula, the results obtained for the test of the 7/8 inch pipe at 90°F. are shown in Table 13.

TABLE 13.

Mean Velocity Ft. per Sec.	Actual Head Lost per 100 ft. Experimental.	Head Lost per 100 ft. from Theoretical Formula. Using Actual Diameter	Head Lost per 100 ft. from Theoretical Formula. Using Nominal Diameter	Difference Nom. Theor. & Exp. %
2	31·5	31·9	35·4	+ 12·4
4	63·5	63·8	70·7	+ 11·3
6	96·0	95·6	106	+ 10·4
8	128	127·5	141	+ 10·2
10	162	159	177	+ 9·1
12	198	191	212	+ 7·1
14	234	223	248	+ 6·0

It will be seen that when the actual diameter is used the calculated and experimental results are in very close agreement. Using/

Using the nominal diameter however, the calculated results are higher than those obtained experimentally, the difference ranging from 6 per cent to 12.4 per cent.

The amount of the error in any particular case will of course depend on the variation of the diameter from its nominal value.

ERRORS DUE TO USING NOMINAL DIAMETERS.

From equation (10) the head lost per 100 ft. of pipe under stream line flow conditions is

$$h = \frac{3200 \nu V}{32.2 d^2}$$

For particular values of viscosity and velocity this may be written

$$h = \frac{K}{d^2}$$

and if d_1 = actual diameter

d_2 = nominal diameter.

$$\frac{h_1}{h_2} = \frac{d_2^2}{d_1^2}$$

$$\text{and} \quad h_2 = h_1 \times \left(\frac{d_1}{d_2}\right)^2 \quad \text{- - - - - - - - - (14).}$$

The differences obtained by using the nominal diameters are shown in Table 14.

TABLE 14./

TABLE 14.

Nominal Diameter d_2	Actual Diameter d_1	$\frac{d_1}{d_2}$	$\left(\frac{d_1}{d_2}\right)^2$	$h_2 = h_1 \times \left(\frac{d_1}{d_2}\right)^2$	Difference h_2 and h_1 %
1"	1"	1	1	h	0
7/8"	$\frac{59"}{64}$	1.052	1.11	1.11 h	+ 11
3/4"	$\frac{53"}{64}$	1.104	1.22	1.22 h	+ 22
5/8"	$\frac{41"}{64}$	1.024	1.05	1.05 h	+ 5
3/8"	$\frac{7"}{16}$	1.166	1.36	1.36 h	+ 36

14. DIRECT READING DIAGRAM.

Since the diameter of a solid-drawn steel pipe cannot be guaranteed to a definite size, the nominal diameter is commonly used in calculation.

In such pipes it is usual to allow a manufacturing tolerance of 5 per cent (plus or minus) on the thickness of the material.

In pipes of small diameter this may cause a considerable difference between the nominal and actual internal diameters.

For a 1-inch pipe the probable maximum difference is about 7 per cent, and this gradually increases as the pipes become smaller to about 30 per cent. for a 3/8 inch pipe. It will probably most often happen in the smaller pipes that the nominal diameter is less than the actual and, if this is the case, the calculated loss of head will be greater than that which will actually occur in the pipe.

Since there is this uncertainty there would appear to be no great objection to using the approximate formula already given in equation (12).

From this formula, $h = .000278 \frac{VT}{d^2}$ we have

$$T = \frac{h\, d^2}{.000278\, V}$$

and if $d = 1$ inch

$h = 100$ feet.

$$T = \frac{100}{.000278 \times 144\, V} = \frac{2500}{V} \quad \text{- - - - - - (15).}$$

From equation (15) may be calculated the values of Redwood viscosity at which a loss of head of 100 feet occurs in a pipe 100 feet in length for various velocities and, since T is proportional to h the viscosities corresponding to other heads are readily obtainable.

These values have been tabulated in Table 15.

TABLE 15./

TABLE 15.

Mean Velocity Ft./Sec.	Head lost per 100 ft. of pipe - Feet of Water.											
	100	200	300	400	500	600	700	800	900	1000	1100	1200
	Redwood Viscosity - Seconds.											
2	1250	2500	3750	5000	6250	7500	8750	10,000	11250	12500	13750	15000
4	625	1250	1875	2500	3125	3750	4375	5000	5625	6250	6875	7500
6	417	834	1250	1670	2080	2500	2920	3330	3750	4170	4580	5000
8	313	625	938	1250	1560	1875	2185	2500	2810	3130	3440	3750
10	250	500	750	1000	1250	1500	1750	2000	2250	2500	2750	3000
12	208	416	625	834	1040	1250	1460	1670	1870	2080	2290	2500
14	178	357	536	714	892	1070	1250	1430	1610	1780	1960	2140
16	156	312	468	625	781	936	1090	1250	1410	1560	1720	1870
18	139	278	416	556	695	833	973	1110	1250	1390	1530	1670
20	125	250	375	500	625	750	875	1000	1125	1250	1360	1500
22	114	227	341	454	568	681	795	908	1022	1135	1250	1360
24	104	208	312	416	521	625	728	833	937	1040	1145	1250
26	96.1	192	289	385	481	576	674	770	865	961	1060	1150
28	89.3	179	268	357	446	536	625	715	804	893	982	1070
30	83.3	167	250	334	416	500	583	676	750	833	916	1000

When the Redwood viscosities are plotted against velocities the curves shown in Fig. 22 are obtained.

CRITICAL VELOCITIES CALCULATED USING NOMINAL DIAMETER.

From equation (6)

$$\frac{Vd}{\nu} = 1200$$

$$V = \frac{1200\,\nu}{d}$$

and if d is in inches

$$V = \frac{14400\,\nu}{d} \qquad (16)$$

Using the nominal diameters of the pipes and the viscosities at which the experiments were carried out the corresponding critical velocities have been calculated. These are given in Table 16.

TABLE 16.

Redwood Viscosity	Kinematic Viscosity	Nominal Diameter - Inches.					
		1	7/8	3/4	5/8	1/2	3/8
		Critical Velocity - Feet per Second.					
1341	.003743	53.9	61.6	71.9	86.2	107.8	143.5
905	.002538	36.5	41.7	48.7	56.8	73.0	97.4
680	.001904	27.4	31.3	36.5	43.8	54.8	73.1
634	.001780	25.6	29.2	34.1	41.0	51.2	68.4
474	.001327	19.1	21.8	25.4	30.6	38.2	51.0
463	.00130	18.8	21.4	25.0	30.0	37.5	50.0
338	.0009452	13.6	15.5	18.1	21.8	27.2	36.3
330	.0009259	13.4	15.3	17.8	21.4	26.7	35.6
260	.0007285	10.5	12.0	14.0	16.8	21.0	28.0
249	.0006941	10.0	11.4	13.3	16.0	20.0	26.7
198	.0005525	7.96	9.1	10.6	12.7	15.9	21.2
185	.0005121	7.4	8.5	9.9	11.6	14.8	19.7
146	.0004003	5.8	6.6	7.7	9.2	11.5	15.4
117	.0003162	4.6	5.2	6.1	7.3	9.1	12.1

These values when plotted give the straight lines shown in Fig. 22.

In order to avoid complication of the diagram, curves of head lost have been drawn at intervals of 100 feet only, but by drawing the diagram to a larger scale additional curves may be put in, giving heads to any required degree of accuracy for any desired range of viscosity.

From this diagram it is possible to read off directly the value of head lost per 100 ft. of pipe at any particular values of velocity and viscosity. It is also seen at once, for any diameter of pipe, if the condition of flow is stream line or turbulent.

If the point lies above the critical velocity line for any particular diameter, the flow is stream line and, if below, it is turbulent.

Further since the diagram is plotted for a pipe of one inch diameter and since the curves are plotted from the equation

$$h = .000278 \frac{VT}{d^2}$$

it follows that the value of h for any other diameter D at the same velocity and viscosity, is obtained by dividing the value for the one inch pipe, obtained from the diagram, by D^2, D being in inches.

COMPARISON OF RESULTS FROM DIAGRAM WITH THEORETICAL AND EXPERIMENTAL VALUES.

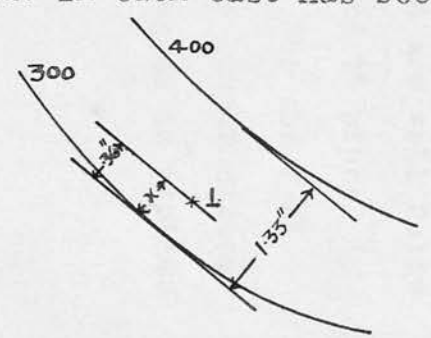
In order to compare the results obtained by the use of the diagram with those obtained by the theoretical formula and by experiment, five points have been taken at wide intervals over the diagram.

These points have been selected at viscosities corresponding to those used in the tests and, where the velocities lie within the range of the experiments the experimental values of head lost are given. These results are recorded in Table 17.

TABLE 17.

Nominal Diameter.					1"	7/8"	3/4"	5/8"	3/8"
Actual Diameter D					1"	59/64"	53/64"	41/64"	7/16"
Point No.	Velocity	Redwood Viscosity		D^2	1	0.85	0.686	0.41	0.195
1	12	680	Head lost per 100 ft of pipe.	Graph.	327	385	477	798	1710
				Theor.	327	386	477	800	1705
				Exp.	321	381	476	828	1670
2	4	1341		Graph	213	251	310	520	1120
				Theor.	214	252	313	525	1118
				Exp.	209	253	328	-	-
3	24	330		Graph.	316	372	461	770	1654
				Theor.	318	374	464	776	1659
				Exp.	-	-	-	810	-
4	26	905		Graph.	941	1108	1370	2295	4920
				Theor.	945	1111	1378	2302	4930
				Exp.	-	-	-	-	-
5	12	260		Graph.	127	149	185	310	650
				Theor.	125	147	182	305	640
				Exp.	141	154	181	286	-

The values of head lost for the one inch pipe are taken from the diagram, but since the curves are drawn at 100 ft. intervals only, the exact value in each case has been obtained by measurement. Thus for point 1.



$$\frac{x}{100} = \frac{0.36}{1.33}$$

$$x = \frac{36}{1.33} = 27$$

and Value of h = 327 ft. per 100 ft. of pipe.

The graphical values for diameters other than one inch are derived from the latter by dividing by the square of the diameter in each case.

The graphical and theoretical results are in close agreement throughout, and the experimental results are fairly close except in three cases.

These are:- 5/8" pipe, Point 3; 1" pipe, Point 5 and 7/8" pipe, Point 5.

It will be noticed that Points 3 and 5 are well below the critical velocity lines for the 5/8" and 1" pipes respectively, while Point 5 is on the line for the 7/8" pipe. This indicates that the flow was no longer stream line so that the formula from which the diagram is plotted no longer holds.

---

15. TURBULENT FLOW.

It has been shown in equation (5) that, for stream line flow

$$\frac{R}{\rho V^2} = \frac{8\nu}{Vd}$$

and since $\nu = \frac{\mu}{\rho}$

$$R = \frac{8\mu V}{d} \quad \text{-----------} \quad (17)$$

Thus for stream line flow the resistance is directly proportional to the viscosity and the velocity and is independent of the density.

When the flow is turbulent, experiment shows that the resistance is more nearly proportional to the square of the velocity.

Stanton* has shown that the index law does not hold except over a very small range of velocities and that the index increases in value as the velocity increases gradually approaching the value 2.0.

When this limiting condition is reached and the resistance becomes proportional to the square of the velocity it will be seen that in the general equation

$$R = \rho V^2 \phi \left(\frac{Vd}{\nu}\right) \quad \text{---------} \quad (2)$$

the function $\frac{Vd}{\nu}$ must have a constant value.

Thus. $R = \rho V^2 \times k.$ ----------- (18)

This indicates that the resistance is now proportional to the density and is independent of the viscosity and of the diameter.

It follows then, that when the flow is turbulent, a change of temperature will have little effect since the change in density/

* Stanton and Pannell. Nat. Phys. Lab. Collected Researches, Vol. XI. 1914.

density is small, and a change in viscosity does not affect the resistance.

It also follows from equation (3)

$$\frac{img}{V^2} = \phi\left(\frac{Vd}{\nu}\right) \text{ - - - - - - - - - - } (3)$$

that if $\phi\left(\frac{Vd}{\nu}\right)$ is a constant then.

$$\frac{img}{V^2} = \text{a constant} \text{ - - - - - - } (19)$$

and since $f = 2 \quad \frac{img}{V^2}$ from equation (8)

f also will have a constant value when the resistance is proportional to the square of the velocity.

Now referring to Fig. 20 it will be seen that beyond the critical velocity the points lie above the theoretical curve. The greatest variation is shown round about the value $\frac{Vd}{\nu}$ = 2500, which is the value given by Stanton as indicating the critical velocity, and beyond this the points lie more regularly along a curve which gradually approaches the horizontal indicating a constant value of $\frac{img}{V^2}$ in the limiting condition.

In Fig. 21 a similar arrangement of points is seen, except that they do not extend so far since the velocities obtainable with the thicker oil were lower than in the first case.

The position and slope of the curve representing turbulent flow varies with the roughness of the surface, and investigations with regard to this have been made by Lees,[1] Lander[2] and Heywood[3] in experiments dealing with the flow of water and steam.

From/

1. Lees. Proc. Roy. Soc. A. Vol. XCI. 1914.
2. Lander. Proc. Roy. Soc. A. Vol. XCII. 1916.
3. Heywood. Proc. Inst. C.E. Vol. 219. 1925.

From the point of view of the present experiments this portion of the curve is relatively unimportant as the velocities used in practice are not likely to exceed the critical velocity.

In Tables 34-41 Vol. II the values of f calculated from the experimental results are shown. The position of the critical velocity is indicated by a horizontal line.

Referring for example to Table 37, and taking the results of a 5/8 inch pipe at temperatures 100°, 110°, 120° and 130°F., it will be seen that immediately beyond the critical velocity the values of f are irregular, but at the higher velocities they tend to become more nearly constant.

For each of the pipes the values of f obtained above the critical velocity have been averaged with the following results.

TABLE 18.

Diameter Oil.	1"	7/8"	3/4"	5/8"	3/8"
	Mean Value of f.				
Heavy Medium	0.0127	0.0108	0.0106	0.0106	0.0105
Extra Heavy	0.0127	0.0111	0.0106	0.0106	-

It will be seen that the values for the different kinds of oil are in very close agreement. The values for the 1 inch pipe are higher than the others, since it was not possible to obtain velocities beyond 14 ft. per sec. Had it been possible to obtain higher velocities in the other pipes the mean values would of course have been smaller throughout, as the value of f decreases as the velocity increases.

The range of the experiments beyond the critical velocity is not sufficient to justify any discussion of the form of the curve representing turbulent flow.

EXPERIMENTAL RESULTS.

BENDS AND ELBOWS.

It will be seen from Fig. I that the bend and elbow in each test length form parts of a continuous pipe line, and the results recorded from the bends and elbows will therefore be influenced by the conditions obtaining in the pipe.

While the results thus obtained may be expected to give a general indication of the comparative losses in straight pipes, bends and elbows, they must be looked upon as being approximate only, and are not intended to be a rigid investigation with a view to determining a theory of the losses in bends and elbows.

16. BENDS.

Gargoyle D.T.E. Oil. Heavy Medium.

The experimental results on the bends for the various pipe diameters are given in Tables 44-48 Vol. II.

The total head lost has been expressed in feet of water and the corresponding curves are shown in Figs. 23-27.

These diagrams are very similar to those obtained for the straight pipes. At the higher temperatures a distinct change in curvature occurs indicating a change from stream line to turbulent flow.

Gargoyle D.T.E. Extra Heavy Oil.

The results for this oil are shown in Tables 49-53 Vol. II and in Figs. 28-32.

Here again the results are similar to those obtained from the straight pipe, an indication of a change in the character/

character of the flow being seen at the high temperatures.

From these curves the values of head lost at the even velocities have been obtained and are tabulated in Tables 54 and 55 Vol. II.

With the D.T.E. Extra Heavy oil it had not been found possible to carry out satisfactory tests on the 5/8 inch straight pipe at temperatures 70°, 80° and 90°F. owing to the difficulty of temperature control and for the same reason no results were obtained on the 3/8 inch straight pipe.

As the lengths of the bends were comparatively short this difficulty did not arise to the same extent, and it was possible to obtain a certain number of results from the small bends.

On the 5/8 inch bend tests were carried out, up to temperatures of 120°F., but on the 3/8 inch bend the only tests possible were made at 70°F. and 80°F., the maximum velocity being 14 feet per second.

Beyond this velocity pressures became too high for the apparatus and it was found to be extremely difficult to obtain higher temperatures owing to the small quantity of oil flowing through the pipe.

For purposes of comparison the values of head lost per foot length of straight pipe were calculated from the theoretical formula and are given in Tables 42A. and 43.

17. LOSS DUE TO SHOCK IN BENDS.

The total loss which occurs in a bend may be looked upon as consisting of two parts, first the loss due to simple friction in the pipe, and second the loss due to shock. The latter portion is due to the disturbance set up in changing the/

the direction of motion of the fluid, and will include also any loss due to the joint between the straight pipe and the bend.

If h = head lost due to shock.

V = mean velocity.

F = coefficient of resistance.

$$h = F.\frac{V^2}{2g}. \quad - - - - - - - - - \quad (20).$$

The values of the loss due to shock have been calculated and are shown in Tables 56-65 Vol. II.

In each case the friction loss for each velocity was obtained from the results on the straight pipes. From this the corresponding friction loss for the length of bend was calculated and this was thendeducted from the total loss to obtain the head lost in shock.

This was then expressed as an equivalent length of straight pipe and the value of the coefficient of resistance F was also tabulated.

Throughout the whole series of tests it will be seen that the value of F decreases as the velocity increases, the loss due to shock becoming of less importance as the friction loss increases.

For convenience of comparison the values of head lost due to shock, expressed as equivalent lengths of straight pipe have been collected together in Table 19.

TABLE 19.

Values of Equivalent Length of Straight Pipe. Bends.

Red figures indicate turbulent flow approaching bend.

Nominal Diameter Inches.	Mean Velocity Ft. per Sec.	Redwood Viscosity - Seconds.													
		1341	905	680	634	474	463	338	330	260	249	198	185	146	117
		Equivalent length of straight pipe - Feet.													
1	2	.279	.331	.284	.428	.474	.411	.446	.679	.718	.526	.550	.714	.896	.910
	4	.383	.362	.377	.455	.558	.476	.671	.724	.691	.641	.635	.922	.936	.894
	6	.402	.486	.487	.536	.621	.615	.768	.766	.785	.80	.789	.916	.880	.953
	8	.419	.574	.567	.596	.677	.712	.860	.801	.774	.814	.821	.834	.841	.466
	10	.422	.645	.637	.659	.731	.806	.916	.845	.778	.818	.756	.778	.280	.510
	12	.413	.710	.689	.723	.754	.857	.940	.907	.798	.783	.648	.399	.440	.191
	14	.330	.744	.727	.774	.750	.880	.863	.951	.784	.748	.268	.191	.242	.050
7/8	2	.198	.460	.333	.554	.455	.586	.476	.484	.375	.591	.588	.938	.846	.818
	4	.221	.536	.436	.556	.603	.622	.614	.722	.625	.717	.806	1.0	.910	.894
	6	.236	.562	.508	.591	.661	.711	.708	.788	.820	.858	.964	1.10	1.0	.890
	8	.251	.571	.561	.644	.708	.827	.836	.885	.980	.938	1.03	1.12	.982	.865
	10	.268	.568	.658	.697	.782	.895	.914	.962	1.01	.952	1.01	1.11	.951	.767
	12	.285	.590	.690	.785	.858	.964	.965	.997	1.0	.990	.984	1.07	.90	.903
	14	.301	.602	.774	.876	.950	.994	1.03	1.01	.990	1.02	1.0	1.05	.913	.831
	16	.322	.604	.840	.950	1.06	1.04	1.07	1.04	.970	1.04	1.0	1.31	.828	.716
3/4	2	.434	.444	.520	.527	.630	.491	.487	.87	.893	.852	1.0	.910	.813	.890
	4	.390	.505	.628	.605	.690	.907	.718	1.03	1.0	1.04	1.17	1.11	1.04	1.18
	6	.40	.583	.732	.764	.814	.957	.898	1.16	1.21	1.17	1.27	1.28	1.32	1.42
	8	.420	.641	.818	.817	.924	1.05	1.09	1.23	1.43	1.31	1.39	1.42	1.43	1.48
	10	.452	.679	.902	.881	1.05	1.11	1.23	1.31	1.53	1.39	1.47	1.53	1.60	1.44
	12	.502	.750	.987	.938	1.16	1.19	1.37	1.37	1.59	1.47	1.49	1.61	1.70	1.42
	14	.550	.740	1.05	.996	1.24	1.25	1.47	1.40	1.60	1.52	1.53	1.69	1.22	1.61
	16	.617	.761	1.12	1.05	1.33	1.30	1.54	1.44	1.58	1.53	1.49	1.53	1.29	1.46
	18	.681	.798	1.14	1.11	1.39	1.39	1.59	1.46	1.55	1.52	1.39	1.69	1.39	1.36
	20	.773	.840	1.12	1.17	1.43	1.46	1.58	1.48	1.49	1.48	1.66	1.69	1.34	1.33

TABLE 19 (Cont^{d}.)

<u>Values of Equivalent Length of Straight Pipe.</u> <u>Bends.</u>

Nominal Diameter Inches.	Mean Velocity Ft. per Sec.	Redwood Viscosity - Seconds.													
		1341	905	680	634	474	463	338	330	260	249	198	185	146	117
		Equivalent length of straight pipe - Feet.													
	2	.008	.051	.085	.096	.234	.289	.194	.574	.761	.289	-	.750	.593	.596
	4	.017	.028	.045	.189	.316	.373	.220	.614	.849	.319	-	.706	.618	.815
	6	.093	.051	.101	.214	.40	.404	.328	.676	.866	.372	-	.862	.875	1.08
	8	.136	.068	.118	.247	.454	.416	.411	.706	.888	.476	-	.871	1.0	1.07
5/8	10	.160	.085	.174	.279	.515	.427	.465	.712	.900	.563	-	.901	1.05	1.08
	12	.188	.122	.229	.328	.554	.448	.557	.732	.898	.621	-	.945	1.18	1.06
	14	.235	.161	.279	.358	.621	.479	.650	.749	.919	.724	-	.966	1.30	.866
	16	.266	.204	.318	.40	.676	.501	.732	.761	.946	.785	-	1.0	1.15	.559
	18	.322	.247	.357	.473	.732	.534	.764	.766	.964	.822	-	.916	.757	.546
	20	.402	.370	.414	.530	.793	.571	.882	.786	.982	.858	-	.810	.583	.465
	2	.295	.056	.238	-	.160	-	.377	-	-	.667	-	.870	-	-
	4	.307	.078	.286	-	.277	-	.456	-	-	.769	-	.980	-	-
	6	.307	.145	.336	-	.396	-	.538	-	-	.858	-	1.0	-	-
	8	.318	.185	.372	-	.547	-	.581	-	-	.882	-	1.06	-	-
3/8	10	.332	.216	.462	-	.635	-	.652	-	-	.971	-	1.08	-	-
	12	-	-	.544	-	.704	-	.678	-	-	1.03	-	1.14	-	-
	14	-	-	.618	-	.758	-	.762	-	-	1.12	-	1.21	-	-
	16	-	-	.709	-	.826	-	.838	-	-	1.22	-	1.26	-	-
	18	-	-	.784	-	.874	-	.952	-	-	1.31	-	1.36	-	-
	20	-	-	.850	-	.934	-	1.06	-	-	1.40	-	1.46	-	-

On examining this table it will be found that the values for the lowest velocity (2 ft. per sec) are somewhat erratic. This is probably due to the fact that the bend was situated at some distance from the pressure gauge and as the pressure difference was small, it took an appreciable time for the mercury gauge to settle owing to the resistance in the connecting pipes, so that, in some cases the true reading was probably not obtained.

Taken generally however, the results show that for any one velocity the values gradually increase as the viscosity decreases and also that for any particular viscosity the values increase as the velocity increases.

This holds up to the point which indicates turbulent flow in the oil approaching the bend. Beyond the point the values are erratic.

18. EFFECT OF RADIUS OF CURVATURE.

In constructing the test length of pipe it was decided to form the bend simply by bending the pipe as this is the method commonly used in practice with this class of piping. With a view to obtaining an indication of the effect of the radius of curvature no special radius was stated, it being left to the makers to use what was thought to be a suitable radius for each size of pipe.

The ratio of the radii are given in Table 1.

The effect of this ratio is shown in Fig. 33.

The results at a velocity of 4 feet per second were selected as, at this velocity, the flow is likely to be steady throughout the whole series of pipes and the values of head lost for five viscosities varying from 1341 seconds to 185 seconds have been plotted.

In all cases the minimum value of head lost is obtained in/

in the 5/8 inch pipe in which

$$\frac{\text{Radius of Bend}}{\text{Radius of Pipe}} = 11.24.$$

As the ratio increases to 11.7, the value for the 1-inch pipe, the loss of head increases rapidly and it is probable that the minimum value of loss of head occurs at a lower ratio than that indicated in the diagram.

For practical purposes the convenient value 10 may be taken as being the best value for this ratio. This value was also obtained by Schoder* in experiments on the flow of water in 6-inch pipes.

EQUIVALENT LENGTHS IN TERMS OF PIPE DIAMETERS.

The equivalent lengths given above may be conveniently stated in terms of pipe diameters with sufficient accuracy for purposes of practical design.

These are shown in Table 20, the nearest whole number being stated in each case.

The results have been arranged in order of viscosity, irrespective of the kind of oil used, and it will be seen that where the viscosities are of approximately the same value, as for example at 338 Seconds (D.T.E. Heavy Medium) and 330 Seconds (D.T.E. Extra Heavy) the results are in very close agreement for the first three pipes. The results on the 5/8 inch pipe are not quite so uniform, but this was to be expected as the conditions were much more difficult.

It was not possible to carry out the complete set of experiments with the 3/8 inch pipe owing to the high pressures necessary and the difficulty of temperature control.

* Schoder. Proc. Am. Soc. C.E. 1908.

TABLE 20.

Values of Equivalent Lengths in terms of Pipe Diameters. Bends.

Red figures indicate turbulent flow approaching bend.

Nominal Diameter Inches.	Mean Velocity Ft. per Sec.	Redwood Viscosity.													
		1341	905	680	634	474	463	338	330	260	249	198	185	146	117
		Equivalent length of straight pipe - Diameters (feet).													
1	2	2	4	4	5	6	5	5	8	8	6	7	9	11	11
	4	4	4	5	5	7	6	8	9	8	8	8	11	11	11
	6	5	6	6	6	7	7	9	9	9	10	10	11	11	11
	8	5	7	7	7	8	9	10	10	9	10	10	10	10	6
	10	5	8	8	8	9	10	11	10	9	10	9	9	4	6
	12	5	8	8	9	9	10	11	11	10	9	8	5	5	2
	14	5	9	9	9	9	11	11	11	9	9	3	2	3	1
7/8	2	3	6	4	7	6	6	6	6	5	8	8	12	11	11
	4	3	7	6	7	8	8	8	9	8	9	11	13	12	12
	6	3	7	7	8	9	9	9	10	11	11	13	14	13	12
	8	3	7	7	8	9	11	11	12	13	12	13	15	13	11
	10	3	7	9	9	10	12	12	13	13	12	13	14	12	10
	12	4	8	9	10	11	13	13	13	13	13	13	14	12	12
	14	4	8	10	11	12	13	13	13	13	13	13	14	12	11
	16	4	8	11	12	14	14	14	14	13	13	13	17	11	9
3/4	2	6	6	8	8	9	7	7	13	13	12	15	13	12	13
	4	6	7	9	9	10	13	10	15	15	15	17	16	15	17
	6	6	8	11	11	12	14	13	17	18	17	18	19	19	20
	8	6	9	12	12	13	15	16	18	21	19	20	21	21	21
	10	7	10	13	13	15	16	18	19	22	20	21	22	23	21
	12	7	11	14	14	17	17	20	20	23	21	22	23	25	21
	14	8	11	15	14	18	18	21	20	23	22	22	24	18	23
	16	9	11	16	15	19	19	22	21	23	22	22	22	19	21
	18	10	12	16	16	20	20	23	21	22	22	20	24	20	20
	20	11	12	16	17	21	21	23	22	22	21	24	24	19	19

TABLE 20. (Cont^{d}.)

Values of Equivalent Lengths in terms of Pipe Diameters. Bends.

Nominal Diameter Inches.	Mean Velocity Ft. per Sec.	Redwood Viscosity.													
		1341	905	680	634	474	463	338	330	260	249	198	185	146	117
		Equivalent length of straight pipe - Diameters (feet).													
5/8	2	1	1	2	2	4	5	4	11	14	5	-	14	11	11
	4	1	1	1	3	6	7	4	11	16	6	-	13	12	15
	6	2	1	2	4	8	8	6	13	16	7	-	16	16	20
	8	3	1	2	4	9	8	8	13	17	9	-	16	19	20
	10	3	2	3	5	10	8	9	13	17	11	-	17	20	20
	12	3	2	4	6	10	8	10	14	17	12	-	18	22	20
	14	4	3	5	7	12	9	12	14	17	14	-	18	24	16
	16	5	4	6	8	13	9	14	14	18	15	-	19	22	10
	18	6	5	7	9	14	10	14	14	18	15	-	17	14	10
	20	8	7	8	10	15	11	16	15	18	16	-	15	11	9
3/8	2	8	2	7	-	4	-	10	-	-	18	-	24	-	-
	4	8	2	8	-	8	-	13	-	-	21	-	27	-	-
	6	8	4	9	-	11	-	15	-	-	24	-	27	-	-
	8	9	5	10	-	15	-	16	-	-	24	-	29	-	-
	10	9	6	12	-	17	-	18	-	-	26	-	30	-	-
	12	-	-	15	-	19	-	19	-	-	28	-	31	-	-
	14	-	-	17	-	21	-	21	-	-	30	-	33	-	-
	16	-	-	19	-	23	-	23	-	-	33	-	35	-	-
	18	-	-	21	-	24	-	26	-	-	36	-	37	-	-
	20	-	-	23	-	26	-	29	-	-	38	-	40	-	-

19. ELBOWS.

The experimental results on the elbows for the two kinds of oil are given in Tables 66-75 Vol. II.

The corresponding curves showing the total head lost in each case are plotted in Figs. 34-43.

These diagrams are of the same general character throughout but are different from those obtained with straight pipes and bends.

The curves are steeper and there is no indication, even at the highest temperatures of a change in the character of the flow.

Since there is a sudden change in the direction of flow in passing round a right-angled elbow, there is disturbance even at the lowest velocities and as the velocity increases the loss of head increases rapidly.

The values of head lost at even velocities are given in Tables 76, 77 Vol. II.

20. LOSS DUE TO SHOCK IN ELBOWS.

In Tables 78-87 Vol. II are given the values of head lost due to shock, obtained by deducting the frictional loss corresponding to the length of the elbow from the total loss. The equivalent lengths of straight pipe and the values of the coefficient of resistance are also tabulated.

On comparing these tables with the corresponding ones for the bends, Tables 56-65 Vol. II it is seen that in the elbows the shock losses increase much more rapidly as the velocity increases, the value of the coefficient of resistance becoming approximately constant at the higher velocities.

TABLE 21.

Values of Equivalent Length of Straight Pipe. Elbows.

Red figures indicate turbulent flow approaching elbow.

Nominal Diameter Inches.	Mean Velocity Ft. per Sec.	Redwood Viscosity - Seconds.													
		1341	905	680	634	474	463	338	330	260	249	198	185	146	117
		Equivalent length of straight pipe - Feet.													
1	2	.010	.029	.155	.041	.391	.082	.570	.280	.513	.652	.550	.808	1.02	1.07
	4	.110	.333	.622	.555	.980	.816	1.44	1.20	1.65	1.74	2.0	2.37	2.64	2.89
	6	.326	.648	1.0	1.05	1.42	1.42	2.13	1.95	2.51	2.63	3.05	3.34	3.89	4.16
	8	.553	1.01	1.36	1.46	1.82	1.94	2.64	2.54	3.11	3.26	3.77	3.95	4.40	4.16
	10	.748	1.31	1.71	1.80	2.24	2.38	3.13	3.06	3.58	3.75	4.16	4.36	3.45	3.40
	12	.904	1.56	2.01	2.14	2.60	2.80	3.57	3.48	4.05	4.18	4.21	3.93	3.42	3.37
	14	.988	1.80	2.27	2.46	2.90	3.15	3.86	3.85	4.37	4.57	3.50	3.44	3.32	3.37
7/8	2	.206	.304	.572	.607	.818	.560	1.24	.710	1.21	1.73	1.47	2.0	2.08	2.45
	4	.486	.659	.865	1.02	1.31	1.29	1.81	1.79	2.29	2.46	2.89	3.09	3.71	4.04
	6	.667	.976	1.22	1.47	1.77	1.94	2.42	2.56	3.22	3.28	4.16	4.15	4.97	5.33
	8	.835	1.24	1.63	1.92	2.36	2.61	3.12	3.30	4.11	4.14	5.12	5.14	5.87	6.58
	10	.990	1.48	2.02	2.31	2.94	3.14	3.84	3.94	4.74	4.85	5.76	6.0	6.70	6.31
	12	1.13	1.72	2.42	2.69	3.48	3.63	4.48	4.52	5.39	5.49	6.34	6.76	6.98	5.82
	14	1.28	1.95	2.84	3.07	4.0	4.01	5.14	5.05	5.89	6.12	6.82	7.06	5.80	5.88
	16	1.41	2.14	3.19	3.39	4.44	4.37	5.70	5.52	6.34	6.69	7.0	7.24	5.79	5.90
3/4	2	.025	.093	.165	.108	.352	.145	.564	.316	.464	.888	.364	.901	.875	.890
	4	.105	.245	.352	.360	.627	.481	.883	.636	.965	1.05	1.02	1.41	1.56	1.71
	6	.228	.374	.596	.627	.873	.813	1.19	.950	1.41	1.47	1.51	1.94	2.19	2.51
	8	.332	.529	.814	.911	1.11	1.10	1.50	1.37	1.80	1.84	1.94	2.37	2.75	3.14
	10	.429	.674	.990	1.04	1.35	1.35	1.77	1.66	2.13	2.20	2.31	2.71	3.29	3.39
	12	.521	.821	1.14	1.23	1.56	1.58	2.02	1.90	2.38	2.52	2.68	3.11	3.55	2.91
	14	.608	.958	1.29	1.39	1.75	1.76	2.26	2.12	2.64	2.82	3.07	3.50	3.03	2.65
	16	.70	1.07	1.44	1.53	1.98	1.92	2.50	2.28	2.94	3.11	3.37	3.62	2.81	2.70
	18	.786	1.18	1.60	1.65	2.20	2.09	2.76	2.43	3.24	3.39	3.43	3.37	2.74	2.73
	20	.879	1.26	1.73	1.76	2.40	2.22	3.03	2.54	3.50	3.67	3.28	3.11	2.70	2.70

TABLE 21 (Cont^{d}.)

Values of Equivalent Length of Straight Pipe. Elbows.

Nominal Diameter Inches.	Mean Velocity Ft. per Sec.	Redwood Viscosity - Seconds.													
		1341	905	680	634	474	463	338	330	260	249	198	185	146	117
		Equivalent length of straight pipe - Feet.													
5/8	2	.168	.085	.138	.032	.30	.022	.629	.115	.174	1.02	-	1.22	1.26	1.33
	4	.203	.132	.194	.205	.433	.254	.835	.410	.570	1.22	-	1.51	1.75	1.81
	6	.221	.188	.292	.348	.611	.453	1.03	.687	.916	1.45	-	1.91	2.30	2.25
	8	.252	.257	.382	.461	.760	.615	1.19	.924	1.21	1.61	-	2.20	2.63	2.59
	10	.282	.344	.487	.570	.888	.782	1.33	1.16	1.51	1.87	-	2.35	2.88	2.80
	12	.313	.436	.590	.684	.990	.963	1.46	1.38	1.77	2.06	-	2.69	3.26	2.84
	14	.351	.544	.680	.785	1.10	1.11	1.58	1.56	2.01	2.27	-	2.86	3.48	2.53
	16	.390	.646	.780	.877	1.21	1.26	1.69	1.73	2.24	2.42	-	2.89	3.26	2.10
	18	.440	.754	.877	.994	1.31	1.41	1.78	1.89	2.47	2.54	-	3.12	2.67	1.96
	20	.506	.843	.976	1.11	1.53	1.54	1.93	2.03	2.63	2.63	-	3.09	2.12	1.96
3/8	2	.149	.198	.263	-	.415	-	.338	-	-	.367	-	.189	-	-
	4	.199	.253	.398	-	.608	-	.670	-	-	.736	-	.738	-	-
	6	.238	.294	.484	-	.744	-	.931	-	-	1.07	-	1.22	-	-
	8	.295	.341	.585	-	.855	-	1.15	-	-	1.36	-	1.59	-	-
	10	.357	.372	.683	-	.952	-	1.29	-	-	1.54	-	1.85	-	-
	12	-	.414	.794	-	1.05	-	1.44	-	-	1.69	-	2.07	-	-
	14	-	.489	.896	-	1.17	-	1.58	-	-	1.85	-	2.26	-	-
	16	-	-	.994	-	1.31	-	1.73	-	-	2.03	-	2.40	-	-
	18	-	-	1.14	-	1.45	-	1.88	-	-	2.18	-	2.50	-	-
	20	-	-	1.23	-	1.62	-	2.02	-	-	2.33	-	2.60	-	-

TABLE 22.

Values of Equivalent Length in terms of Pipe Diameters. Elbows.

Red figures indicate turbulent flow approaching elbow.

Nominal Diameter Inches.	Mean Velocity Ft. per Sec.	Redwood Viscosity.													
		1341	905	680	634	474	463	338	330	260	249	198	185	146	117
		Equivalent length of straight pipe - Diameters (Feet).													
1	2	1	1	2	1	5	1	7	4	6	8	7	10	12	13
	4	1	4	8	7	12	10	18	18	20	22	25	29	33	36
	6	4	8	12	13	17	18	26	24	31	33	38	41	48	52
	8	7	13	17	18	23	24	33	31	39	40	47	49	55	52
	10	9	16	21	22	28	30	39	38	44	46	52	54	43	42
	12	10	19	25	27	32	35	44	43	50	52	52	49	42	42
	14	12	22	28	30	36	39	48	48	54	57	43	43	41	42
7/8	2	3	4	7	8	11	7	16	9	16	23	19	26	27	32
	4	6	9	11	13	17	17	24	23	30	32	38	40	48	53
	6	9	13	16	19	23	25	32	33	42	43	54	54	65	69
	8	11	16	21	25	31	34	41	43	53	54	67	67	77	86
	10	13	19	26	30	38	41	50	51	62	63	75	78	87	82
	12	15	22	31	35	45	47	58	59	70	71	83	88	91	76
	14	17	25	37	40	52	52	67	66	77	80	89	92	76	77
	16	19	28	41	44	58	57	74	72	83	87	91	94	75	77
3/4	2	1	1	2	2	5	2	8	5	7	13	5	13	13	13
	4	2	4	5	5	9	7	13	9	14	15	15	20	23	25
	6	3	5	9	9	13	12	17	14	20	21	22	28	32	36
	8	5	8	12	13	16	16	22	20	26	27	28	34	40	46
	10	6	10	14	15	20	20	26	24	31	32	34	39	48	49
	12	8	12	17	18	23	23	29	28	35	37	39	45	52	42
	14	9	14	19	20	25	26	33	31	38	41	45	51	44	39
	16	10	16	21	22	29	28	36	33	43	45	49	53	41	39
	18	11	17	23	24	32	30	40	35	47	49	50	49	40	40
	20	13	18	25	26	35	32	44	37	51	53	48	45	39	39

TABLE 22 (Cont$^{d.}$)

Values of Equivalent Length in terms of Pipe Diameters. Elbows.

Nominal Diameter Inches.	Mean Velocity Ft. per Sec.	Redwood Viscosity - Seconds.													
		1341	905	680	634	474	463	338	330	260	249	198	185	146	117
		Equivalent length of straight pipe - Diameters (Feet).													
5/8	2	3	2	3	1	6	1	11	2	3	19	-	23	24	25
	4	4	3	4	4	8	5	16	8	11	23	-	28	33	34
	6	4	4	5	7	11	9	19	13	17	27	-	36	43	42
	8	5	5	7	9	14	12	21	17	23	30	-	41	49	49
	10	5	6	9	11	17	15	25	22	28	35	-	44	54	53
	12	6	8	11	13	19	18	27	26	33	39	-	50	61	53
	14	7	10	13	15	21	21	30	29	38	43	-	54	65	48
	16	7	12	15	17	23	24	32	32	42	45	-	54	61	39
	18	8	14	16	19	25	27	33	35	46	48	-	58	50	37
	20	9	16	18	21	29	29	36	38	49	49	-	58	40	37
3/8	2	4	5	7	-	11	-	9	-	-	10	-	5	-	-
	4	5	7	11	-	17	-	18	-	-	20	-	20	-	-
	6	7	8	13	-	20	-	25	-	-	29	-	33	-	-
	8	8	9	16	-	23	-	32	-	-	27	-	44	-	-
	10	10	10	19	-	26	-	35	-	-	42	-	51	-	-
	12	-	11	22	-	29	-	40	-	-	46	-	57	-	-
	14	-	13	25	-	32	-	43	-	-	51	-	62	-	-
	16	-	-	27	-	36	-	47	-	-	56	-	66	-	-
	18	-	-	31	-	40	-	52	-	-	60	-	69	-	-
	20	-	-	34	-	44	-	55	-	-	64	-	71	-	-

The equivalent lengths of straight pipe are shown in Table 21.

These have also been expressed in terms of pipe diameters and are tabulated in Table 22.

At the low velocities the values are irregular owing to the difficulty of measuring the small losses accurately with the existing apparatus but at the higher velocities the results are much more consistent.

Again comparing the results obtained with different kinds of oil at 338 seconds (D.T.E. Heavy Medium) and 330 seconds (D.T.E. Extra Heavy) the results correspond with fair accuracy.

From this table there is no apparent regular variation of the values of equivalent length with variation of the diameters.

The values for the 7/8 inch pipe are consistently high throughout, and as the difference is very marked this pipe was disconnected and the elbow examined. It was found in this case that a quantity of the white lead used in jointing had been squeezed through into the elbow, causing a very considerable additional obstruction, thus increasing the losses at this point. It was unfortunately impossible to repeat the results after this obstruction had been removed as the oil had been returned to the Vacuum Oil Company.

SUMMARY OF RESULTS.

In designing pipe lines for the transmission of power by means of oil, the pipes should be made of ample diameter. The loss of head for a given velocity of flow increases very rapidly as the diameter of the pipe decreases with a consequent loss in power and increase of pressure at the pump to maintain a given power at the motor unit.

By the provision of pipes of suitable diameter the velocities may be kept below the critical velocity of the oil, in which condition the losses may be accurately determined without experiment provided the viscosity of the oil is known.

Where temperature conditions are likely to vary it is essential that calculations be made for the oil at its lowest temperature otherwise owing to the increase in viscosity it may become difficult, if not impossible, to maintain the necessary power under certain conditions.

Owing to the conditions of manufacture, the actual diameters of solid drawn pipes rarely coincide with the nominal diameters.

On this account calculations made using the nominal diameter will only give an approximate forecast of the losses in the pipes.

LOSS OF HEAD.

For such conditions the loss of head is determined with sufficient accuracy by the formula:-

$$h = .000278 \frac{VT}{d^2} \qquad \text{where}$$

h = loss of head per 100 ft. of pipe in feet of water.

d = diameter of pipe in feet.

V = mean velocity in ft. per sec.

T = Redwood viscosity in seconds.

From this formula a direct reading diagram may be conveniently /

conveniently plotted which is applicable to any diameter of pipe for wide ranges of viscosity and velocity.

CRITICAL VELOCITY.

Under practical conditions where there is vibration and there are bends, joints and other causes of disturbance the critical velocity may be estimated by the formula:-

$$V_c = 0.00336 \frac{T}{d} \quad \text{where}$$

T = Redwood viscosity in seconds

d = diameter of pipe in feet

V_c = critical velocity in feet per sec.

The formulae given above have been stated in foot units but for use with small pipes it may be convenient to express the diameter in inches. The formulae then become.

$$h = .000278 \times 144 \frac{VT}{d^2} = 0.040 \frac{VT}{d^2}$$

and

$$V_c = .00336 \times 12 \frac{T}{d} = 0.0403 \frac{T}{d}. = 0.040 \frac{T}{d} \text{ (practically).}$$

where T = Redwood viscosity in seconds.

V = mean velocity in ft. per sec.

V_c = critical velocity in ft. per sec.

d = diameter of pipe in inches.

RIGHT-ANGLED BENDS.

For minimum loss of head the radius of curvature of the bend should be 10 times the radius of the pipe. In manufacture strict adherence to the specified radius should be insisted upon as a comparatively small difference in radius may cause a considerable increase in the resistance.

Compared with the frictional resistance the loss due to shock in a bend is small and while the former increases rapidly as the velocity increases, the latter becomes relatively unimportant.

ELBOWS.

Elbows should be avoided in design. They cause disturbance of the flow at even the lowest velocities and, while at low velocities the loss due to shock is small compared with the frictional loss, at higher velocities the shock loss increases very rapidly and becomes the more important.

CALCULATION OF TOTAL LOSS.

In calculating the total loss of head due to resistance in a system of piping the length, including the lengths round bends and elbows should first be determined.

The loss due to shock, for a bend, expressed as an equivalent length of pipe is then obtained from the tables and this, multiplied by the total number of bends gives the length of pipe to be added to allow for the loss in the bends.

Similarly the equivalent length for the elbows is calculated.

From the total length so obtained the total loss of head may be calculated by means of the formula for straight pipes.

C O N C L U S I O N.

In conclusion the author desires to express his indebtedness to Professor Sir T. Hudson Beare for permitting the work to be carried out in the Engineering Department of the University of Edinburgh, for his assistance in providing part of the apparatus and for his interest in the work; to the Vacuum Oil Company Ltd., for providing the oil and for carrying out the necessary viscosity tests; to Messrs John Hastie & Co., Ltd., Greenock, for the use of the Hele-Shaw pump; to Messrs Dewrance & Co., Ltd., London, for providing the Bourdon gauges used in the tests and to Mr Charles Patterson of the Engineering Department, University of Edinburgh, for his valuable assistance in preparing tracings of the drawings.
